International Journal of Research in Advent Technology, Vol.6, No.8, August 2018
E-ISSN: 2321-9637
Available online at www.ijrat.org

Properties of One Modulo N Mean Graphs

D. Muthuramakrishnan[1] K. Bhagyasri[2]
[1]*Associate Professor, Department of Mathematics, National College, Trichy*
[2]*Research Scholar, Department of Mathematics, National College, Trichy*

Abstract - The concept of mean labeling was introduced by Somasundaram and Ponraj [11]. Swaminathan and Sekar [12] introduced the concept of one modulo three graceful labeling. Jayanthi and Maheswari [8] introduced one modulo three mean labeling of graphs. It is further studied by Gayathri and Prakash in ([2]-[5]). In [9], we have introduced the concept of one modulo N mean labeling of graphs and obtained its labeling for some family of graphs. It is further studied by us in [10]. In this paper, we investigate the properties of one modulo N mean graphs.

Keywords - mean labeling; one modulo three mean labeling ;one modulo N mean labeling (OMNML); one modulo N mean graph (OMNMG).

1. INTRODUCTION

By a graph $G = (V(G), E(G))$ with p vertices and q edges we mean a simple, connected and undirected graph. In this paper a brief summary of definitions and other information is given in order to maintain compactness. The term not defined here are used in the sense of Harary [7].

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. Somasundaram and Ponraj [11] have introduced the notion of mean labeling of graphs. Swaminathan and Sekar [12] introduced the concept of one modulo three graceful labeling. Jayanthi and Maheswari [8] introduced one modulo three mean labeling. It is further studied by Gayathri and Prakash in ([2]-[5]). In [9], we have introduced the concept of one modulo N mean labeling and obtained results for some family of graphs. It is further studied by us in [10]. In this paper, we investigate the properties of one modulo N mean graphs.

2. MAIN RESULTS

Definition 2.1

A Graph $G = (p, q)$ is said to be **one modulo three mean graph** if there is a function f from the vertex of G to the set $\{0, 1, 3, 4, 6, 7, ..., 3q - 3, 3q - 2\}$ with f is one-one and f induces a bijection f^* from the edge set of G to the set $\{1, 4, 7, 10, ..., 3q - 5, 3q - 2\}$ where $f^*(uv) = \left\lceil \frac{f(u) + f(v)}{2} \right\rceil$ and the function f is called as **one modulo three mean labeling** of G. Here $f^*(uv) \equiv 1(\text{mod}\,3)$ for every edge uv in G and $\lceil\,\rceil$ represents the ceil function.

Definition 2.2

A Graph $G = (p, q)$ is said to be **one modulo N mean graph (OMNMG)** if there is a function f from the vertex of G to the set $\{0, 1, N, N + 1, 2N, 2N + 1…., N(q - 1), N(q - 1) + 1\}$ with f is one-one and f induces a bijection f^* from the edge set of G to the set $\{1, N + 1, 2N + 1, 3N + 1, …, N(q - 2) + 1, N(q - 1) + 1\}$ where $f^*(uv) = \left\lceil \frac{f(u) + f(v)}{2} \right\rceil$ and the function f is called as **one modulo N mean labeling(OMNML)** of G where N is any positive integer. Here, $f^*(u,v) \equiv 1(\text{mod } N)$ for every edge uv is G and $\lceil\,\rceil$ represents the ceil function.

Example 2.3

The graph shown in Figure 2.1 is a one modulo 4 mean graph.

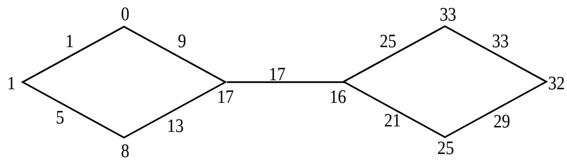


Figure 2.1: One modulo 4 mean graph

Observation 2.4

(i) $N(q - 1) + 1 \equiv 1(\text{mod } N)$ for all q and N.
(ii) If N is even then $N(q - 1) + 1$ is odd for all q.
(iii) If q is odd then $N(q - 1) + 1$ is odd for all N.
(iv) If q is even and N is odd then $N(q - 1) + 1$ is even.
(v) $N(q - 1) + 1 \equiv \begin{cases} 1(\text{mod } 2N) & q \text{ is odd} \\ N + 1(\text{mod } 2N) & q \text{ is even} \end{cases}$
(vi) If N is odd then $N(q - 1)$ is even for all q.
(vii) If q is odd then $N(q - 1)$ is even for all N.
(viii) If q is even and N is odd, then $N(q - 1)$ is odd.
(ix) $N(q - 1) \equiv \begin{cases} 0(\text{mod } 2N) & q \text{ is odd} \\ N(\text{mod } 2N) & q \text{ is even} \end{cases}$
(x) If $x \equiv 0(\text{mod } N)$ then $x \equiv 0(\text{mod } 2N)$ or $N(\text{mod } 2N)$.
(xi) If $x \equiv 1(\text{mod } N)$ then $x \equiv 1(\text{mod } 2N)$ or $N + 1(\text{mod } 2N)$.

Proof

(i) to (iv) and (vi)-(viii) are routine to check. (ix) follows from (v).

We first prove (x).

Let $x \equiv 0 (\text{mod } N)$ then $x = Nk$ for some k.

Case 1: N is even.

Then x is even.

Sub case 1: k is even.

Then let $k = 2l$ for some l.

Therefore $x = Nk = N2l \equiv 0(\text{mod } 2N)$

Sub case 2: k is odd.

Let $k = 2l + 1$ for some l.

Then $x = N\,k = N(2l + 1) = N2l + N \equiv N(\text{mod } 2N)$.

Case 2: N is odd.

Sub case 1: x is even.

Then k is even and hence $k = 2l$ for some l.

$\therefore\ x = N\,k = N2l \equiv 0(\text{mod } 2N)$.

Sub case 2: x is odd.

Then k is odd and hence $k = 2l + 1$ for some l.

$\therefore\ x = N\,k = N(2l + 1) \equiv N(\text{mod } 2N)$.

Hence the property (x) is true.

We now prove (xi)

Let $x \equiv 1(\text{mod } N)$ then $x - 1 \equiv 0(\text{mod } N)$

by (x),

$x - 1 \equiv 0(\text{mod } 2N)$ or $x - 1 \equiv N(\text{mod } 2N)$

Therefore $x \equiv 1(\text{mod } 2N)$ or

$x \equiv N + 1(\text{mod } 2N)$

Thus the property (xi) is proved.

We now prove the property (v).

Case 1: q is odd.

Then $x = N(q - 1) + 1$ is odd.

$q - 1$ being even, let $q - 1 = 2l$ for some l.

Therefore $x = N(2l) + 1 \equiv 1(\text{mod } 2N)$

Case 2: q is even.

$q - 1$ being odd, $q - 1 = 2l + 1$ for some l.

Therefore $x = N(q - 1) + 1 = N(2l + 1) + 1 =$

$2lN + N + 1 \equiv N + 1 \ (\text{mod } 2N)$

Thus the property (v) follows.

Property 2.5

If a graph G is a one modulo N mean graph for any N then 0 and 1 are adjacent vertex labels.

Proof

The induced edge label 1 can only be obtained by the adjacent vertex label pair (0, 1). Therefore 0 and 1 are ought to be vertex labels.

Property 2.6

If a graph $G = (p, q)$ is a one modulo N mean graph for any N then $N(q - 1)$ and $N(q - 1) + 1$ are ought to be the adjacent vertex labels.

Proof

By Definition 2.2, the number $N(q - 1) + 1$ has to be an edge label. In order to have $N(q - 1) + 1$ as an induced edge label, the only possible adjacent vertex label pair is $(N(q - 1), N(q - 1) + 1)$. Hence $N(q - 1)$ and $N(q - 1) + 1$ are adjacent vertex labels.

Theorem 2.7

Let $G = (p, q)$ be a one modulo N mean graph with one modulo N mean labeling f. Let t be the number of edges whose one end vertex label is even and other is odd. The $\sum_{v\in v(G)} d(v)f(v) + t = q(N(q-1)+2)$ where $d(v)$ denotes the degree of the vertex v.

Proof

$$\sum_{v\in v(G)} d(v)f(v) = 2\left[\sum_{vu\in E(G)} f^*(uv) - \frac{t}{2}\right]$$

$= 2[1 + (N + 1) + (2N + 1) + \ldots.$
$(N(q - 1) + 1)] - t$

$$= 2\left[\frac{(\frac{N(q-1)}{N}+1)}{2}(N(q-1)+2)\right] - t$$

$= q(N(q - 1) + 2) - t$

Therefore $\sum_{v\in v(G)} d(v)f(v) + t = q(N(q-1)+2)$

Corollary 2.8

If $G = (p, q)$, $q \geq 2$ is a one modulo N mean graph with one modulo N mean labeling f then $\sum_{v\in v(G)} d(v)f(v) \geq Nq$.

Proof

By Theorem 2.7,

$$\sum_{v\in v(G)} d(v)f(v) + t = q(N(q-1)+2)$$

$$\sum_{v\in v(G)} d(v)f(v) = q(N(q-1)+2) - t$$

$$\sum_{v\in v(G)} d(v)f(v) \geq q(N(q-1)+2q-2q$$

(since $t \leq q \leq 2q$)

$$\sum_{v\in v(G)} d(v)f(v) \geq qN(q-1)$$

$\sum_{v\in v(G)} d(v)f(v) \geq Nq$ (since $q \geq 2$).

Property 2.9

Let $G = (p, q)$ be a l–regular one modulo N mean graph with l even. Let t be the number of edges whose one end vertex label is even and other is odd then t is even.

Proof

By Theorem 2.7,

$$\sum_{v\in v(G)} d(v)f(v)+t = q(N(q-1)+2) \qquad \ldots (1)$$

G being l–regular,

$$l\sum_{v\in v(G)} f(v)+t = q(N(q-1)+2)$$

$$t = q(N(q-1)+2) - l\sum_{v\in v(G)} f(v) \qquad \ldots (2)$$

If q is even then l is even implies right hand side of (2) is even.

If q is odd then $N(q-1)+2$ is even and l is even, it follows that right hand side of (2) is even. Hence t is even.

Corollary 2.10

If $G = (p, q)$ is a 2-regular one modulo N mean for any N then t is even.

Theorem 2.11

Let $G = (p, q)$ be a 2-regular one modulo N mean graph for any N. Let f be a one modulo N mean labeling of G. Let $x \in \{0, 1, 2N, 2N+1, \ldots. N(q-1), N(q-1)+1\} = S\backslash f(V(G))$.

Let t be the number of edges whose one end vertex label is even and the other end is odd then $x = \frac{t}{2} + \frac{(q-1)(N-1)}{2}$.

Proof

By Corollary 2.10, t is even say $2m$. By Theorem 2.7,

$$\sum_{v\in v(G)} d(v)f(v)+t = q(N(q-1)+2)$$

$$2\sum_{v\in v(G)} f(v)+2m = q(N(q-1)+2)$$

$$\sum_{v\in v(G)} f(v)+m = \frac{q(N(q-1)+2)}{2} \quad (0+2N+\ldots+N(q-1)+$$

$$(1+2N+1 \ldots N(q-1)+1) - x + m = \frac{q(N(q-1)+2)}{2}$$

$$\left(\frac{q+1}{4}\right)N(q-1)+\left(\frac{q+1}{4}\right)(N(q-1)+2)-x+m = \frac{q(N(q-1)+2)}{2}$$

$$\left(\frac{q+1}{4}\right)\left[2N(q-1)+2\right]-x+m = \frac{q(N(q-1)+2)}{2}$$

$$\left(\frac{q+1}{2}\right)\left[N(q-1)+1\right]-\frac{q(N(q-1)+2)}{2} = x-m$$

$$\left(\frac{q+1}{2}\right)(N(q-1)+1)-\frac{q}{2}(N(q-1)+2) = x-m$$

$$\frac{qN(q-1)+N(q-1)+q+1-qN(q-1)-2q}{2} = x-m$$

$$\frac{N(q-1)-q+1}{2} = x-m$$

$$\frac{N(q-1)-(q-1)}{2} = x-m$$

$$\frac{(q-1)(N-1)}{2} = x-m$$

$$x = m + \frac{(q-1)(N-1)}{2}$$

$$x = \frac{t}{2}+\frac{(q-1)(N-1)}{2}$$

Thus $x = \frac{t}{2}+\frac{(q-1)(N-1)}{2}$.

Property 2.12

Let $G = (p, q)$ be a one modulo N mean graph for any N.

(i) If q is odd then 0, 1, $N(q-1)+1$ cannot be the vertex labels of the cycle C_3 contained in G for all N.

(ii) If N is even then 0, 1, $N(q-1)+1$ cannot be the vertex labels of the cycle C_3 contained in G for all q.

(iii) If q is even and N is odd then 0, 1, $N(q-1)$ cannot be the vertex labels of the cycle C_3 contained in G.

(iv) If N is even then 1, $N(q-1)$, $N(q-1)+1$ cannot be the vertex labels of the cycle C_3 contained in G for all q.

(v) If q is odd then, 1, $N(q-1)$, $N(q-1)+1$ cannot be the vertex labels of the cycle C_3 contained in G for all N.

(vi) If q is even and N is odd then 0, $N(q-1)$, $N(q-1)+1$ cannot be the vertex labels of the cycle C_3 contained in G.

Proof

Let G be a one-modulo N mean graph for any N. Let a_0, a_1, a_2 be the vertices of a cycle C_3 contained in G as in Figure 3.2.

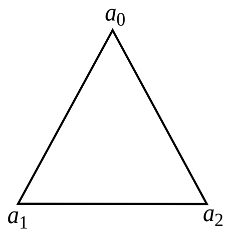
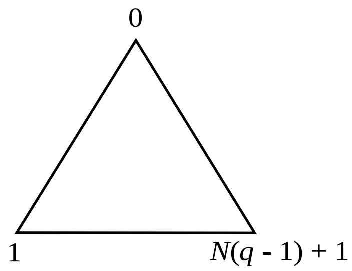


Figure 3.2

(i) q is odd. Then by observation 2.4 (iii), $N(q-1)+1$ is odd.

Suppose 0, 1, $N(q-1)+1$ be the vertex labels of the cycle C_3 contained in G. Then without loss of generality, assume

$f(a_0) = 0$, $f(a_1) = 1$, $f(a_2) = N(q-1)+1$



In this case, the induced edge labels are:

$f^*(a_0a_1) = 1$

$f^*(a_1a_2) = \frac{1+N(q-1)+1}{2} = \frac{N(q-1)+2}{2}$

$f^*(a_0a_2) = \frac{N(q-1)+1+0+1}{2} = \frac{N(q-1)+2}{2}$

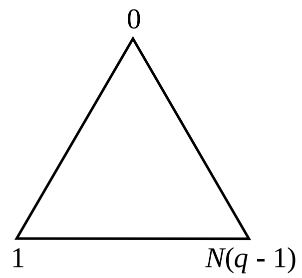
a contradiction to G is a one modulo N mean graph. Hence (i) follows.

(ii) Let N be even. Then by observation 2.4 (ii), $N(q-1)+1$ is odd for all q.

Suppose 0, 1, $N(q-1)+1$ are the vertex labels of the cycle C_3 contained in G then by following the argument used in the proof (i) yields a contradiction to G is a one modulo N mean graph. Hence (ii) is proved.

(iii) Let q be even and N is odd. Then observation 2.4 (viii), $N(q-1)$ is odd.

Suppose 0, 1, $N(q-1)$ are the vertex labels of the cycle C_3 contained in G. Then without loss of generality, assume $f(a_0) = 0$, $f(a_1) = 1$, $f(a_2) = N(q-1)$.



In this case, the induced edge labels are:

$f^*(a_0a_1) = 1$

$f^*(a_1a_2) = \frac{N(q-1)+1}{2}$

$f^*(a_0a_2) = \frac{N(q-1)+0+1}{2} = \frac{N(q-1)+1}{2}$

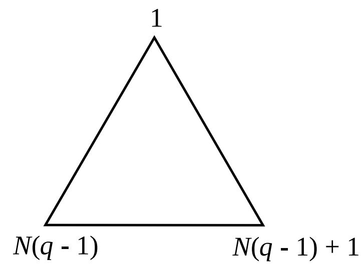
a contradiction to G is a one modulo N mean graph. Thus (iii) is proved.

(iv) Let N be even. Thus by observation 2.4, $N(q-1)+1$ is odd for all q.

Suppose 1, $N(q-1)$, $N(q-1)+1$ are the vertex labels of the cycle C_3 contained in G.

Assume without loss of generality,

$f(a_0) = 1$, $f(a_1) = N(q-1)$, $f(a_2) = N(q-1)+1$



In this case, the induced edge labels are:

$f^*(a_0a_1) = \frac{1+N(q-1)+1}{2} = \frac{N(q-1)+2}{2}$

$f^*(a_1a_2) = \mathrm{N}(q-1)+1$

$f^*(a_0a_2) = \frac{N(q-1)+2}{2}$

a contradiction to G is an one modulo N mean graph. Thus (iv) is proved.

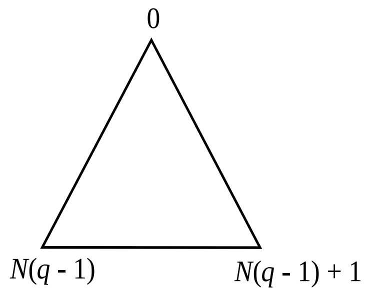
(v) Let q be odd. By observation 2.4 (iii), $N(q-1)+1$ is odd for all q.

Suppose 1, $N(q-1)$, $N(q-1)+1$ are the labels of the vertices of the cycle C_3 contained in G then by following the argument used in the proof of (iv) yields a contradiction to G is a one modulo N mean graph. Hence (v) is proved.

(vi) Let q be even and N be odd. Then by observation 2.4 (viii), $N(q-1)$ is odd.

Suppose 0, $N(q-1)$, $N(q-1)+1$ are the vertex labels of the cycle C_3 contained in G, assume without loss of generality

$f(a_0) = 0$, $f(a_1) = N(q-1)$, $f(a_2) = N(q-1)+1$



In this case, the induced edge labels are:

$f^*(a_0a_1) = \frac{0+N(q-1)+1}{2} = \frac{N(q-1)+1}{2}$

$f^*(a_0a_2) = \frac{0+N(q-1)+1}{2} = \frac{N(q-1)+1}{2}$

a contradiction to G is a one modulo N mean graph. Hence the theorem.

Corollary 2.13

The cycle C_3 is not a one modulo N mean graph for any N.

REFERENCES

[1] J.A. Gallian, "A Dynamic Survey of Graph Labeling", *The Electronic Journal of Combinatorics,* DS6, 2016.

[2] B. Gayathri and V. Prakash, Necessary conditions for one modulo three mean labeling of graphs, *International Journal of Engineering Sciences, Advanced Computing and Bio-Technology,* Vol. 6, No. 2, April-June 2015, 41-61.

[3] B. Gayathri and V. Prakash, One modulo three mean labeling of some special graphs, *Mathematical Sciences International Research Journal,* Vol. 4, Issue 2 (2015), 434-460.

[4] B. Gayathri and V. Prakash, One modulo three mean labeling of some family of trees, *International Journal of Engineering Sciences, Advanced Computing and Bio-Technology,* Vol. 6, No. 4, October-December 2015, 141-161.

[5] B. Gayathri and V. Prakash, One modulo three mean labeling of disconnected graphs, *Jamal Academic Research Journal: Interdisciplinary*, Special Issue, February 2016, 21-26.

[6] R. Gopi, A study on different kinds of mean labeling, Ph.D. Thesis, Bharathidasan University, Trichy, February 2013.

[7] F. Harary, 1998, "Graph Theory", *Narosa Publishing House Reading*, New Delhi.

[8] P. Jeyanthi and A. Maheswari, "One modulo three mean labeling of graphs", *Amer. J. App. Math. And Stat.,* Vol. 2, No. 5, 2014, 302-306.

[9] D. Muthuramakrishnan and K. Bhagyasri, "On one modulo *N* mean graphs", Accepted for publication in *IJSRR* – To appear.

[10] D. Muthuramakrishnan and K. Bhagyasri, "One modulo *N* mean labeling of graphs", Accepted for publication in *IJSRR* – To appear.

[11] S. Somasundaram and R. Ponraj, "Mean labeling of graphs", *National Academy of Science Letters",* Vol. 26 (2003), 210-213.

[12] V. Swaminathan and C. Sekar, "Modulo three graceful graphs", *Proceed. National Conference on Mathematical and Computational Models,* PSG College of Technology, Coimbatore, 2001, 281-286.

[13] S. K. Vaidya and Lekha Bijukumar, Some new families of mean graphs, Journal of Mathematics Research, 2(3) (2010) 169-176.

[14] S.K. Vaidya and Lekha Bijukumar, New mean graphs, *International Journal of Mathematical Combinatorics,* 3, (2011) 107-113.